MATHEMATICS SOLUTION
(CBCGS SEM – 4 MAY 2019)
BRANCH – IT ENGINEERING

1a) Find the remainder when 2^{50} is divided by 7. **(05)**

Ans. We know, $2^3=8=7x1+1$

$\therefore 2^3 = 8 \equiv 1 (\text{mod } 7)$

$\therefore (2^3)^{16} \equiv 1^{16} (\text{mod } 7)$

$\therefore 2^{48} \equiv 1 (\text{mod} 7)$

$\therefore 2^{48} x\ 2^2 \equiv 1 \text{ x } 2^2 \ (\text{mod } 7)$

$\therefore 2^{50} \equiv 4 (\text{mod } 7)$

Hence, the remainder when 2^{50} is divided by 7 is 4.

1b) The probability distribution function of random variable X is **(05)**

X	0	1	2	3	4	5	6
P (x = x)	k	3k	5k	7k	9k	11k	13k

Find (i)k; (ii)p(x≤4); (iii)p(x≤ 4); (iv)p(3<x<6); (v)p(3<x≤6).

Ans. For any probability mass function, $\sum_{i=-\infty}^{\infty} p_i = 1$

$\therefore p(0)+p(1)+p(2)+p(3)+p(4)+p(5)+p(6)=1$

$\therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$\therefore 49k = 1$

$\therefore k = \frac{1}{49}$

The p.m.f is

X	0	1	2	3	4	5	6
P (x)	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

∴ p (x<4)= p(0)+p(1)+p(2)+p(3)

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49}$

$= \frac{16}{49}$

∴ p(x≤ 4) = p(0)+p(1)+p(2)+p(3)+p(4)

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} + \frac{9}{49}$

$= \frac{25}{49}$

∴p(3<x<6) = p(4)+p(5)

$= \frac{9}{49} + \frac{11}{49}$

$= \frac{20}{49}$

P(3<x≤ 6) = p(4)+p(5)+p(6)

$= \frac{9}{49} + \frac{11}{49} + \frac{13}{49}$

$= \frac{33}{49}$

Hence,

$K = \frac{1}{49}$; $p(x<4) = \frac{16}{49}$; $p(x \le 4) = \frac{25}{49}$; $p(3 < x < 6) = \frac{20}{49}$; $p(3<x\le 6) = \frac{33}{49}$

1c) Calculate the value rank correlation coefficient from the following data regarding score of 6 students in physics & chemistry test. **(05)**

Marks in physics: 40, 42, 45, 35, 36, 39

Marks in chemistry: 46, 43, 44, 39, 40, 43

Ans. Let X and Y denote Marks in statistics and Accountancy respectively.

X	Y	R_1	R_2	$d_1=R_1-R_2$	d_1^2
40	46	3	1	2	4
42	43	2	3.5	-1.5	2.25
45	44	1	2	-1	1.00
35	39	6	6	0	0
36	40	5	5	0	0
39	43	4	3.5	0.5	0.25
				Total =	7.5

Here $n = 6$, $m_1=2$

$\therefore$spearmen's Rank correlation Coefficient is $R = 1 - \frac{6}{n(n^2-1)}\{\Sigma d_1^2+\frac{1}{12}(m_1^3-m_1)\}$

$= 1 - \frac{6}{6(6^2-1)}\left\{7.5 + \frac{1}{12}(2^3 - 2)\right\}$

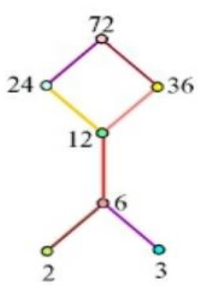
$= 1 - \frac{1}{35}\left\{7.5 + \frac{1}{12} x\ 6\right\}$

$= 0.7714$

The value of rank correlation coefficient = 0.7714

1d) Draw the Hasse diagram of poset A = {2,3,6,12,24,36,72} under the relation of divisibility. Is it Lattice? **(05)**

Ans. A = {2, 3, 6, 12, 24, 36, 72}, Under the relation of divisibility, the Hasse Diagram is



We know the relation of divisibility is a partial order relation.

∴ Set (A,R) is a poset.

Consider,

∧	2	3	6	12	24	36	72
2	2	-	2	2	2	2	2
3	-	3	3	3	3	3	3
6	2	3	6	6	6	6	6
12	2	3	6	12	12	12	12
24	2	3	6	12	24	12	24
36	2	3	6	12	12	36	36
72	2	3	6	12	24	36	72

We, observe, from the above table, that some pair of elements of A do not have a GLB (greatest lower bound) ∈A. Hence (A,R) is not a lattice.

2a) If x is a Poisson variate such that P(x = 2)= 9P(x = 4) + 90P(x = 6) then find mean of x. **(06)**

Ans. For Poisson distribution, $P\ (X\ =\ x\ ) = \frac{e^{-m} m^x}{x!}$

Given, P(x = 2)=9 P(x = 4)+90 P(X = 6)

$\therefore \frac{e^{-m} m^2}{2!} = \frac{9e^{-m} m^4}{4!} + \frac{90e^{-m} m^6}{6!}$

$\therefore \frac{e^{-m} m^2}{2} = \frac{3e^{-m} m^4}{8} + \frac{e^{-m} m^6}{8}$

$\therefore \frac{e^{-m} m^2}{2} = \frac{e^{-m} m^2}{8}(3m^2 + m^4)$

$\therefore 4 = 3m^2 + m^4$

$\therefore m^4 + 3m^2 - 4 = 0$

$\therefore m^2 = -4$ or $m^2 = 1$

$\therefore m = \pm 2i$ or $m = \pm 1$

Assuming m is real and positive, m = 1

Hence, for the given Poisson distribution,

∴ Mean = m = 1 and Variance = m =1

2b) Consider (3,4) parity check code. For each of the following received words determine whether an error will be detected? (i) 0010;(ii) 1001; (iii) 1101; (iv) 1111. (06)

Ans. Let H be a (3,4) parity check code

Let H = $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

We define syndrome of 'r' as $s=rH^T$, Where 'r' is the received word.

If the syndrome 's' is not all zeros, then an error has been detected. If it is All zeros, then that the codeword can be assume as correct.

We use binary addition for matrix operation

+	0	1
0	0	1
1	1	0

(i) Received code (r) = (0010)

$$\therefore\ s = rH^T = [0\ 0\ 1\ 0] \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [0\ 1\ 0]$$

As the syndrome 's' is not all zeros, the error can be detected.

(ii) Received Code (r) - (1001)

$$\therefore s = rH^T = [1\ 0\ 0\ 1] \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 0\ 0]$$

As the syndrome 's' is not all zeros, the error can be detected.

(iii) Received Code (r) = (1101)

$$\therefore\ s = rH^T = [\ 1\ 1\ 0\ 1] \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [\ 0\ 0\ 0]$$

As the syndrome 's' is all zeros, the error cannot be detected.

(iv) Received Code (r) - (1111)

$$\because s = rH^T = [\ 1\ 1\ 1\ 1]\ x \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [\ 0\ 1\ 0]$$

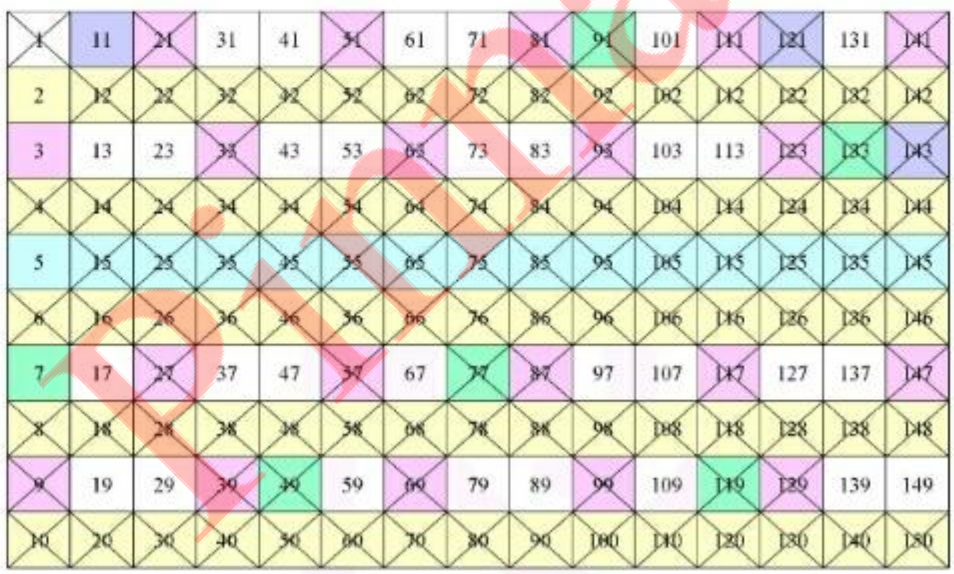
As the syndrome 'S' is not all zeros, the error can be detected.

2c) Using sieve of Eratosthenes, find the prime number upto 150. (04)

Ans. Sieve of Eratosthenes

First, we write all numbers from 1 to 150

We consider all multiples of prIme numbers less than $\sqrt{150}$ = 12.24≈12 i.e. 2, 3, 5, 7, 11 and cancel them to get the prime number up to 150.



Hence, prime number upto 150 are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71, 73, 79, 83, 89, 97, 101, 103,107, 109, 113, 127, 131, 137, 139, 149.

2d) What is the remainder when following sum is divided by 4?$1^5 + 2^5 + 3^5 + \ldots + 100^5$. (04)

Ans. $1^5+2^5+3^5+4^5 \pmod 4 = 5^5+6^5+7^5+8^5 \pmod 4 = 9^5+10^5+11^5+12^5 \pmod 4$ = so on i.e we will get 25 such sets.

So if we find what $1^5+2^5+3^5+4^5 \pmod 4$ is, we can simply multiply it by 25 to get the final result.

$1^5 \pmod 4 = 1$

$2^5 \pmod 4 = 0$

$3^5 \pmod 4 = 3$

$4^5 \pmod 4 = 0$

So $1^5+2^5+3^5+4^5 \pmod 4 = (1+0+3+0) \pmod 4$

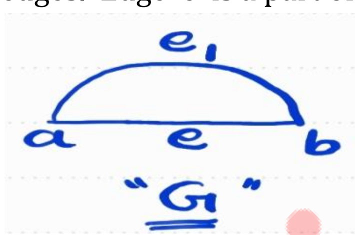
$= 4 \pmod 4$

$= 0$ (remainder)

Hence final answer is 0x25=0

3a) Prove that a graph 'G' remains connected after removing an edge 'e' from 'G' iff 'e' is in some circuit of G. (06)

Ans._A circuit is a walk that starts and ends at a same vertex and Contains no repeated edges. Edge 'e' is a part of a circuit of G. Let 'a' and 'b' be the endpoints of edge e.



∴vertices 'a' and 'b' are the part of circuit G.

∴ There exist two paths from 'a' to 'b'. one of this path includes the edge 'e'.

On removal of edge 'e' , there exists a path from a to b that does not includes the edge 'e'.

Hence , graph 'G' remains connected.

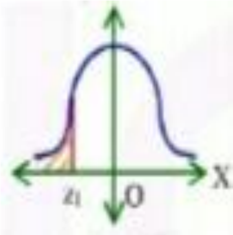
3b) Marks obtained by students in an examination follow normal distributions. (06)
If 30% of students got below 35 marks and got above 60 marks. Find mean and standard deviation.

Ans. Let the mean and standard derivation be 'm' and 'σ'.

Let X denote marks obtained by a student in an examination.

Part I:

Let SNV corresponding to $x_1 = 35$ be z_1

P(x < 35) = 30% 

$\therefore$ P $(z<z_1) = 0.30$

$\therefore$ 0.5 – Area between 'z = 0' to 'z=-z', is 0.30

$\therefore$ Area between 'z = 0' to 'z= -z', is 0.20

From z-table, $-z_1 = 0.5244$ i.e, $z_1 = -0.5244$

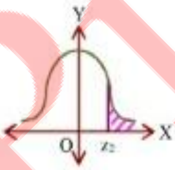
But, $z_1 = \frac{x_{1-m}}{\sigma}$

$\therefore -0.5244 = \frac{35-m}{\sigma}$

$\therefore m-0.5244\sigma = 35 \longrightarrow (1)$

Part II:

Let SNV corresponding to $x_2 = 60$ be z_2.



$P(x > 60) = 10\%$

$\therefore p(z>z_2) = 0.10$

$\therefore$ 0.5 – Area between 'z = 0' to 'z=z_2' is 0.10

$\therefore$ Area between 'z = 0' to 'z=z_2' is 0.40

From z-table, $z_2 = 1.2816$

But, $z_2 = \frac{x_{2-m}}{\sigma}$

$\therefore 1.2816 = \frac{60-m}{\sigma}$

$\therefore \text{m} + 1.2816\sigma = 60 \quad \longrightarrow (2)$

Solving (1) & (2) simultaneously we get, $\text{m} = 42.2593$ and $\sigma = 13.8431$

Hence,

Mean Marks $= \text{m} \approx 42$

Standard Deviation $= \sigma = 13.8431$

Variance $= \sigma^2 = 13.8431^2 = 191.6314$

3c) Investigate the association between the darkness of eye colour in father and son from the following data: (08)

	Colour of Father's eyes			
Colour of son's eyes		Dark	Not Dark	Total
	Dark	48	90	138
	Not Dark	80	782	862
	Total	128	872	1000

Ans.

Observed Frequency (O)	Expected Frequency (E)	$x^2 = \frac{(o-E)^2}{E}$
48	18	50,0000
90	120	7.5000
80	110	8.1818
782	752	1.1968
	Total	66.8786

Step I:

Null Hypothesis (H_0) : There is no association between the darkness of eye colour in father and son.

Alternative Hypothesis (H_a) : There is association between the darkness of eye colour in father & son. (Two tailed test).

Step 2:

LOS = 5% (Two tailed test)

Degree of Fredom = (r -1) (c-1)

= (2 – 1) (2 – 1)

= 1

∴ Critical value $(x_a{}^2) = 3.841$

Step 3: Test Statistic

$$x_{cal}{}^2 = \sum \frac{(O-E)^2}{E} = 66.8786$$

Step 5: Decision

Since $x_{cal}{}^2 > x_a{}^2$, H_0 is rejected.

∴ There is association between the darkness of eye colour in father and son.

4a) Using Euclid's Algorithm find x and y satisfying the following : (06)
gcd (-306 , 657) =306x +657y.

Ans. Property of GCD : (a ,b) = (- a , b) (a ,-b)= (-a,-b) = ($|a||b|$)

∴gcd (-306,657) = gcd (306 , 657)

Part I: Let a = 306 and b = 657 , Using Euclid Algorithm,

1	657= 2 × 306 +45	∴ b= 2 × a + 45 ∴ b – 2a = 45
2	306 = 6 × 45 +36	∴ a = 6 × (b-2a)+36 ∴a = 6b – 12a +36 ∴ 13a – 6b = 36
3	45 = 1 × 36 +9	∴b – 2a = 1 x (13a-6b)+9 ∴ b - 2a = 13a – 6b +9 ∴ 7b – 15a =9⟶(1)
4	36 = 1 × 9 +0	

∵ gcd (- 306, 657) = gcd (306, 657) = 9

Part II :

Given, 306 x + 657y = gcd (-306, 637)

$\therefore$ ax + by = 9 $\longrightarrow$ (2)

Comparing (1) and (2), x = -15 and y = 7

i.e., x_0 = - 15 and y_0 = 7 is one solution of 306 x + 657y = gcd (-306, 637)

other solutions are $x = x_0 + \left[\frac{b}{d}\right]$t and y = $y_0 - \left[\frac{a}{d}\right]$t where 't' is arbitrary & d = gcd of a & b i.e. d = (a,b)= 9

$\therefore x$ = -15 + $\left(\frac{657}{9}\right)$t and y = 7 - $\left(\frac{306}{9}\right)$t

$\therefore$ other solutions are $x = -15 + 73t \ \ and \ y = 7 - 34t$

4b) Let L = {1, 2, 3, 5, 6, 10, 15, 30} with divisibility relation. Then show that L is a complimented lattice. (06)

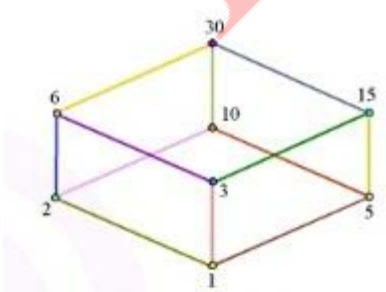
Ans. Given L = {1, 2, 3, 5, 6, 10, 15, 30}, which is a set of Division of 30.

$\therefore$ D_{30} = L = {1, 2, 3, 5, 6, 10, 15, 30}

R is the relation 'is divisible by'

R = {(1, 1),(1, 2), (1, 3),(1, 5),(1, 6),(1, 10),(1,15),(1,30),(2, 2),(2, 6), (2,10), (2, 30),(3, 3),(3, 6),(3, 15), (3, 30),(5, 5),(5,10),(5, 25),(5, 30), (6, 6),(6, 30), (10, 10), (10, 30), (15, 15), (15,30), (30, 30)}

The Hasse diagram is



We know the relation of divisibility is a partial order relation. $\therefore set\ (D_{30,}R)$is a poset.

Consider,

∨	1	2	3	5	6	10	15	30
1	1	2	3	5	6	10	15	30
2	2	2	6	10	6	10	30	30
3	3	6	3	15	6	30	15	30
5	5	10	15	5	30	10	15	30
6	6	6	6	30	6	30	30	30
10	10	10	30	10	30	10	30	30
15	15	30	15	15	30	30	15	30
30	30	30	30	30	30	30	30	30

∨	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

From the two table we observe that every pair of elements of D_{30} has a LUB (least upper bound) and GLB (greatest lower bound). Also, each LUB and GLB$\in D_{30}$

Hence, (D_{30},R) is a lattice.

By definition of a complement, a ∨ $\bar{a}= 1$ and a ∧ $\bar{a} = 0$ i.e, a ∨ $\bar{a}= 30$ and a ∧ $\bar{a} = 1$

From the above two tables we observe, the complements of elements of set A are

Elements	1	2	3	5	6	10	15	30
Complements	30	15	10	6	5	3	2	1

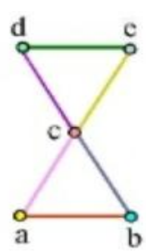
∴ complement of each elements exists. ∴ L is a complimented Lattice.

4c) Give an example of a graph which has (08)

(i) Eulerian circuit but not a Hamiltonian circuit
(ii) Hamiltonian circuit but not an Eulerian circuit
(iii) Both Hamiltonian circuit and Eulerian circuit.
(iv) None of Hamiltonian circuit and Eulcerian circuit.

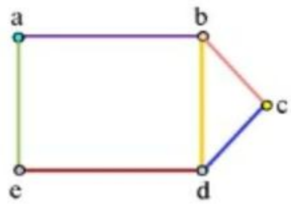
Ans:

(i) Eulerian circuit but not a Hamiltonian circuit



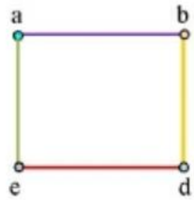
All the vertices are of even degree. Hence by theorem there is Eulerian circuit.
Eulerian circuit : abcdeca
The circuit is not Hamiltonian because there is no circuit which contains all the vertices only once.

(ii) Hamiltonian circuit but not an Eulerian circuit



All the vertices can be traversed only once. Hence there is Hamiltonian circuit.
Hamiltonian circuit : abcdea
The degree of vertices b & d are odd. Hence there is no Eulerian circuit.

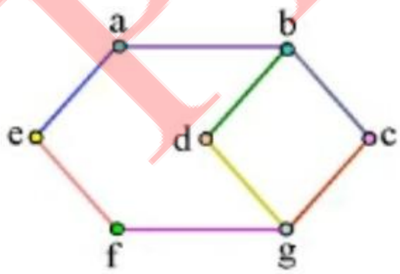
(iii) Both Hamiltonian circuit and Eulerian circuit.



Degree of all vertices are even. Hence there is an Euler circuit abdea.

All the vertices can be traversed only once. Hence there is Hamiltonian circuit.
Hamiltonian circuit : abdea

(iv) None of Hamiltonian circuit and Eulcerian circuit.



The circuit is not Hamiltonian because there is no circuit which contains all the vertices only once.
The degree of vertices b & g are odd. Hence there is no Eulerian circuit.

5a) Fit Binomial Distribution to the following data. **(06)**

x	0	1	2	3	4
Frequency	**12**	**66**	**109**	**59**	**10**

Ans.

X	f	fx	Theoretical frequency
0	12	0	17.42≈17
1	66	66	66.75≈67
2	109	218	95.91≈ 96
3	59	177	61.25≈ 61
4	10	40	14.67 ≈ 15
Total	256	501	256

$$\text{Mean}=\frac{\Sigma f_i x_i}{\Sigma f_i}=\frac{501}{256}=1.9570$$

But, for binomial distribution, mean = np

$\therefore 1.9570 = 4\,p$

$\therefore p = 0.4893$

$\therefore q = 1 - p = 1\text{-}0.4893 = 0.5107$

$n = 4$ and $N = 256$

$\therefore P(X = x) = {}^{n}C_x p^x q^{n-x} = {}^{4}C_x \times (0.4893)^x \times (0.5107)^{4-x}$

$\therefore$ Theoretical frequency = N × p (X = x)

$= 256 \times {}^{4}C_x \times (0.4893)^x \times (0.5107)^{4-x}$

5b) Nine items of a sample had the following values:45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of 9 items differ significantly from the assumed population means 47.5 ? **(06)**

Ans. n=9(<30, so it is small sample)

Step1:

Null Hypothesis (H_0) : $\mu = 47.5$(i.e, sample mean does not differ significantly from the assumed population mean 47.5)

Alternative Hypothesis (H_0) : $\mu \neq 47.5$ (i.e, sample mean differ significally from the assumed population mean 47.5) (two tailed test)

Step 2:

LOS = 5% (Two tailed test)

Degree of Freedom = n – 1=9-1 = 8

∴ Critical value (t_a) = 2.306

Step 3:

Values (x_1)	$d_i = x_i - 47$	d_i^2
45	-2	4
47	0	0
50	3	9
52	5	25
48	1	1
47	0	0
49	2	4
53	6	36
51	4	16
Total	19	95

$\bar{d} = \frac{\Sigma d_1}{n} = \frac{19}{9} = 2.1111$

$\therefore \bar{x} = a + \bar{d} = 47 + 2.11 = 49.11$

Since sample is small, $\delta = \sqrt{\frac{\Sigma d_i{}^2}{n} - \left[\frac{\Sigma d_i}{n}\right]^2}$

$= \sqrt{\frac{95}{9} - \left(\frac{19}{9}\right)^2}$

$= 2.4696$

Standard Error S.E $= \frac{s}{\sqrt{n-1}}$

$= \frac{2.4696}{\sqrt{8}}$

$= 0.8731$

Step 4: Test Statistic

$t_{cal} = \frac{\bar{x}-\mu}{S.E.}$

$= \frac{49.1111 - 47.5}{0.8731}$

$= 1.8452$

Step 5: Decision

Since $|t_{cal}| < t_a$, H_0 is accepted.

∴ sample mean does not differ significantly from the assumed population mean 47.5.

5c) Solve : x =1(mod3) , x= 2(mod5) , x=3(mod7) **(08)**

Ans:

$a_1 = 1 \qquad m_1 = 3$

$a_2 = 2 \qquad m_2 = 5$

$a_3 = 3 \qquad m_3 = 7$

$M = m_1.m_2.m_3$

$M = 3 \times 5 \times 7 = 105$

$M_1 = 5 \times 7 = 35$

$M_2 = 3 \times 7 = 21$

$M_3 = 3 \times 5 = 15$

$M_1x \equiv 1(mod\ m_1)$
$35x \equiv 1(mod\ 3)$
$1 \equiv 35x\ (mod\ 3)$
$1 \equiv 2x(mod3)$
$1 \equiv -x(mod\ 3)$
$x \equiv -1(mod\ 3)$
$x \equiv 2(mod\ 3)$

$M_2x \equiv 1(mod\ m_2)$
$21x \equiv 1(mod\ 5)$
$1 \equiv 21x\ (mod\ 5)$
$1 \equiv 1x(mod\ 5)$
$x \equiv 1(mod\ 5)$

$M_3x \equiv 1(mod\ m_3)$
$15x \equiv 1(mod\ 7)$
$1 \equiv 15x(mod\ 7)$
$1 \equiv 1x\ (mod\ 7)$
$x \equiv 1(mod\ 7)$

By Chinese Remainder Theorem,

$\therefore \quad x \equiv [a_1M_1x_1 + a_2M_2x_2 + a_3M_3x_3] \ modulo \ M.$

$x \equiv [(1)(35)(2) + (2)(21)(1) + (3)(15)(1)] \ modulo \ 105$

$x \equiv 157 \ (mod \ 105)$

$x \equiv 52 \ (mod \ 105)$

$\therefore$ $\boxed{x = 52}$ is one solution . General solution is given by, $\boxed{x = 52 + 105\,k}$ where k is any integer

6a) Given 6y = 5x + 90, 5x = 8y+30, $\sigma_x{}^2$= 16. Find (i) $\bar{x}$ and $\bar{y}$ (ii) r (iii) $\sigma_y{}^2$ (06)

Ans. 6y = 5x + 90

$\therefore y = \frac{5}{6}x + \frac{90}{6}$

$\therefore y = \frac{5}{6}x + 15 \longrightarrow (1)$

And, 5x = 8y + 30

$\therefore$ 8y = 5x – 30

$\therefore y = \frac{5}{8}x - \frac{30}{8} \longrightarrow (2)$

Let $b_1 = \frac{5}{6}$ and $b_2 = \frac{5}{8}$

Since $|b_2| < |b_1|$,

$b_{yx} = b_2 = \frac{5}{8}$ & $b_{xy} = \frac{1}{b_1} = \frac{6}{5} \longrightarrow (3)$

$\therefore$ Equation (2) is regression equation of y on x type.

And equation (1) is regression equation of X on Y type.

From (1) and (2), $\frac{5}{6}x + 15 = \frac{5}{8}x - \frac{30}{8}$

$\therefore \frac{30}{8} + 15 = \frac{5}{8}x - \frac{5}{6}x$

$\therefore \frac{75}{4} = \frac{-5}{24}x$

$\therefore x = -90$

Substitute x = - 90 in (1)

$\therefore$ y = $\frac{5}{6}$(-90)+15 = -60

Now, r = $\pm\sqrt{b_{yx} . b_{xy}}$

$= \pm\sqrt{\frac{5}{8} \times \frac{6}{5}}$ (from 3)

$= \pm 0.8660$

Since, b_{yx} and b_{xy} are both positive. 'r' is positive.

$\therefore$ r = 0.8660 ⟶ (4)

Also, given, σ_x^2=16

$\therefore \sigma_x = 4$ ⟶ (5)

Using, b_{yx} = r $\frac{\sigma_y}{\sigma_x}$

$\therefore \frac{5}{8} = 0.8660 * \frac{\sigma_y}{4}$ (From 3, 4 & 5)

$\therefore \sigma_y = 2.8868$

$\therefore \sigma_y^2 = 8.3333$

Ans. 1) $\bar{x}$ = -90; $\bar{y}$ = -60;

2) r = 0.8660;

3) σ_y^2 = 8.3333.

6b) Prove that set of cube root of unity is a group under multiplication of complex number.

Ans. Let 'ω' be cube root of unity. **(06)**

$\therefore \omega = \sqrt[3]{1}$

$\therefore \omega^3 = 1 \longrightarrow (1)$

$\therefore \ \omega^3 - 1 = 0$

On solving we get, the root as

$$\omega_1 = 1; \ \omega_2 = \frac{-1}{2} + i\frac{\sqrt{3}}{2}; \ \ \omega_3 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$$

Consider, $\omega_2{}^2 = \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right)^2 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}$

$\therefore \omega_3 = \ \omega_2{}^2 \longrightarrow (2)$

Similarly, $\omega_2 = \ \omega_3{}^2 \longrightarrow (3)$

Also, $\omega_2 \times \omega_3 = \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) \times \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 \longrightarrow (4)$

Let G = $\{1, \omega_2, \omega_2{}^2\}$ be the set of set of cube- roots of unity.

The composition table under multiplication of complex number is (using 1, 2, 3, & 4)

x	1	ω_2	$\omega_3 = \omega_2{}^2$
1	1	ω_2	ω_3
ω_2	ω_2	$\omega_2{}^2 = \omega_3$	1
$\omega_3 = \ \omega_2{}^2$	ω_3	1	$\omega_3{}^2 = \omega_2$

From above table, we observe, All the elements belongs to G.

$\therefore$ x is binary operator in G.

G1:

Multiplication of complex number is associative.

$\therefore$ x is associative.

G2:

From table, we observe, first row is same as the header.

$\therefore$ 1 $\in$ G is the identity.

$\therefore$ Identify of x exist.

G3:

From table, we observe, identify elements (i.e.1) is present in each row.

$\therefore 1^{-1} = 1; (\omega_2)^{-1} = \omega_3; (\omega_3)^{-1} = \omega_2$

$\therefore$ Inverse of each element exist and each inverse $\in$ G

$\therefore$ Inverse of x exist.

Hence, set of cube root of unity is a group under multiplication of complex numbers.

6c) Show that $111^{333} + 333^{111}$ is divisible by 7. **(04)**

Ans.

We know , $111 = 16 \times 7 + (-1)$

$\therefore 111 \equiv -1 \pmod 7$

$\therefore 111^{333} \equiv (-1)^{333} \pmod 7$

$\therefore 111^{333} \equiv -1 \pmod 7 \longrightarrow (1)$

Similarly, $333 = 47 \times 7 + (4)$

$\therefore 333 \equiv 4 \pmod 7$

$\therefore\ 333^3 \equiv 4^3 \pmod 7$

$333^3 \equiv 64 \pmod 7$

$333^3 \equiv 1 \pmod 7$

$(333^3)^{37} \equiv 1^{37} \pmod 7$

$(333)^{111} \equiv 1 \pmod 7 \longrightarrow (2)$

$\therefore$ Adding (1) & (2), $111^{333} + 3333^{111} \equiv 1 \pmod 7 + (-1) \pmod 7$

$\therefore 111^{333} + 333^{111} \equiv (1-1) \pmod 7$

$\therefore\ 111^{333} + 333^{111} \equiv 0 \pmod 7$

i.e, Remainder = 0 when $111^{333} + 333^{111}$ is divided by 7.

$\therefore\ 111^{222} + 333^{111}$ is divisible by 7.

6d) Find 5^{-1} (mod23). **(04)**

Ans. 23 is a prime number

By Fermat's little theorem, a^{-1} (mod p) ≡ a^{P-2} (mod p)

a = 5 , p = 23

5^{-1} (mod23) ≡ 5^{23-2} (mod 23)

≡ 5^{21} (mod 23)

≡ $(5^7)^3$ (mod 23)

≡ 17^3 (mod 23)

≡ 14 (mod 23)